





Question 1 continued

Lined area for writing the answer to Question 1.

Q1

(Total 4 marks)



















**Question 3 continued**

Lined writing area for Question 3.

**(Total 8 marks)**

**Q3**



4. Given that

$$y \frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^2 + 5y = 0$$

(a) find  $\frac{d^3 y}{dx^3}$  in terms of  $\frac{d^2 y}{dx^2}$ ,  $\frac{dy}{dx}$  and  $y$ . (4)

Given that  $y = 2$  and  $\frac{dy}{dx} = 2$  at  $x = 0$

(b) find a series solution for  $y$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . (5)

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**Question 4 continued**

Ruled writing area consisting of 26 horizontal lines.







5. (a) Find, in the form  $y = f(x)$ , the general solution of the equation

$$\frac{dy}{dx} + 2y \tan x = \sin 2x, \quad 0 < x < \frac{\pi}{2}$$

(6)

Given that  $y = 2$  at  $x = \frac{\pi}{3}$

(b) find the value of  $y$  at  $x = \frac{\pi}{6}$ , giving your answer in the form  $a + k \ln b$ ,  
where  $a$  and  $b$  are integers and  $k$  is rational.

(4)

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6. The complex number  $z = e^{i\theta}$ , where  $\theta$  is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2 \cos n\theta$$

where  $n$  is a positive integer.

**(2)**

(b) Show that

$$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$$

**(5)**

(c) Hence find all the solutions of

$$\cos 5\theta + 5 \cos 3\theta + 12 \cos \theta = 0$$

in the interval  $0 \leq \theta < 2\pi$

**(4)**

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7. (a) Find the value of  $\lambda$  for which  $\lambda t^2 e^{3t}$  is a particular integral of the differential equation

$$\frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 9y = 6e^{3t}, \quad t \geq 0 \tag{5}$$

(b) Hence find the general solution of this differential equation. (3)

Given that when  $t = 0$ ,  $y = 5$  and  $\frac{dy}{dt} = 4$

(c) find the particular solution of this differential equation, giving your solution in the form  $y = f(t)$ . (5)

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8.

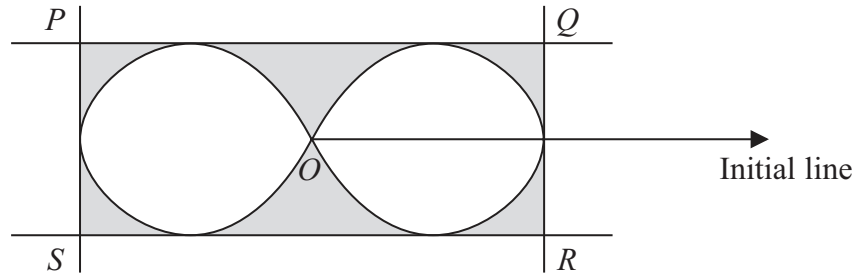


Figure 1

Figure 1 shows a closed curve  $C$  with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \quad \text{where } -\frac{\pi}{4} < \theta \leq \frac{\pi}{4}, \quad \frac{3\pi}{4} < \theta \leq \frac{5\pi}{4}$$

The lines  $PQ$ ,  $SR$ ,  $PS$  and  $QR$  are tangents to  $C$ , where  $PQ$  and  $SR$  are parallel to the initial line and  $PS$  and  $QR$  are perpendicular to the initial line. The point  $O$  is the pole.

- (a) Find the total area enclosed by the curve  $C$ , shown unshaded inside the rectangle in Figure 1. (4)

- (b) Find the total area of the region bounded by the curve  $C$  and the four tangents, shown shaded in Figure 1. (9)

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**Question 8 continued**

Lined area for writing the answer to Question 8.

**(Total 13 marks)**

Q8

Marking box for Q8

**TOTAL FOR PAPER: 75 MARKS**

**END**

